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Calculation of a static potential created by plane fractal cluster

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ABSTRACT

In this paper we demonstrate new approach that can help in calculation of electrostatic potential of a fractal (self-similar) cluster that is created by a system of charged particles. For this purpose we used the simplified model of a plane dendrite cluster [1] that is generated by a system of the concentric charged rings located in some horizontal plane (see Fig. 2). The radiuses and charges of the system of concentric rings satisfy correspondingly to relationships: $r_n = r_0\xi^n$ and $e_n = e_0 b^n$, where n determines the number of a current ring. The self-similar structure of the system considered allows to reduce the problem to consideration of the functional equation that similar to the conventional scaling equation. Its solution represents itself the sum of power-low terms of integer order and non-integer power-law term multiplied to a log-periodic function [5,6]. The appearance of this term was confirmed numerically for internal region of the self-similar cluster ($r_0 \ll r \ll r_{N-1}$), where r_0, r_{N-1} determine the smallest and the largest radiuses of the limiting rings correspondingly. The results were obtained for homogeneously ($b > 0$) and heterogeneously ($b < 0$) charged rings. We expect that this approach allows to consider more complex self-similar structures with different geometries of charge distributions.

1. Introduction

To present time the influence of potential created by the static electric field on a geometric structure of the dendrite growth is intensively investigated [2,4,7,8] – especially in formation of fractal threads and cluster–cluster aggregation under the applied electric field. In paper [10] the electrochemical deposition phenomenon was considered but in solution of the inverse problem we found only a few papers [3,9], where the influence of a self-similar structure on the generated potential was taken into account. The results presented in that paper are qualitative and not complete, because the discrete structure of the fractal considered plays an essential role. In this paper we calculated the electrostatic potential that is generated by a system including N charged rings located in the horizontal plane (Fig. 2). This system was chosen because of one reason. It presents a plane model of dendrite cluster absorbed on a solid substrate (Fig. 1). For this system one can show that the potential created in a certain region of a space, which is determined as the internal region ($r_0 \ll r \ll r_{N-1}$), where r_0, r_{N-1}–the smallest and the largest radiuses of the concentric rings correspondingly, r is a current radius connecting the center of the system with some point P belonging to up or down half-space, satisfies to the scaling functional equation [5] (see Appendix A). Its solution allows to find the nontrivial behaviour of electrostatic potential in the region determined above. We mean the appearance of the term associated with non-integer power-law function combined with log-periodic term. Alongside with this basic result we present the results of numerical calculations that confirm the basic analytical expressions.

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